# Relational Algebra Masterclass 

## Given:

Student(SID, Sname, GPA)
Department(DName, Chair, Building, Room)

Course(DName, CID, CName, Hours)
Enrolled(DName, CID, SID)
where

- DName in Course is a foreign key referencing Department,
- DName,CID in Enrolled is a foreign key referencing Course,
- SID in Enrolled is a foreign key referencing Student, and
- primary keys are underlined.
and the relation states
Student

| SID | Sname | GPA |
| :--- | :--- | :--- |
| 11 | Bush | 3.0 |
| 12 | Cruz | 3.2 |
| 13 | Clinton | 3.9 |
| 22 | Sanders | 3.0 |
| 33 | Trump | 3.8 |

Enrolled

| DName | CID | SID |
| :--- | :--- | :--- |
| CS | 101 | 11 |
| Math | 101 | 11 |
| CS | 101 | 12 |
| CS | 101 | 22 |
| Math | 103 | 33 |
| EE | 102 | 33 |
| CS | 102 | 22 |

Department

| DName | Chair | Building | Room |
| :--- | :--- | :--- | :--- |
| CS | Rubio | Ajax | 100 |
| Math | Carson | Acme | 300 |
| EE | Kasich | Ajax | 200 |
| Music | Costello | North | 100 |

Course

| DName | CID | CName | Hours |
| :--- | :--- | :--- | :--- |
| CS | 101 | Programming | 4 |
| CS | 102 | Algorithms | 3 |
| Math | 101 | Algebra | 3 |
| Math | 103 | Calculus | 4 |
| Music | 104 | Jazz | 3 |
| EE | 102 | Circuits | 3 |

Show how the following relational algebra expression gives the names of all students enrolled in two or more courses.

$$
\pi_{S N a m e}\left(\pi_{S I D}\left(\sigma_{D N a m e \neq D} \text { OR CID} \neq C\left(\rho_{(D, C, S I D)}(\text { Enrolled }) * \text { Enrolled }\right)\right) * \text { Student }\right)
$$

Work from the inside out and play close attention to parentheses and operator-operand binding.
Apply $\rho_{(D, C, S I D)}($ Enrolled $)$, which creates a relation like Enrolled but with Dname and CID renamed:

| D | C | SID |
| :--- | :--- | :--- |
| CS | 101 | 11 |
| Math | 101 | 11 |
| CS | 101 | 12 |
| CS | 101 | 22 |
| Math | 103 | 33 |
| EE | 102 | 33 |
| CS | 102 | 22 |

Then apply $\rho_{(D, C, S I D)}($ Enrolled $) *$ Enrolled, which natural joins the relation created above with Enrolled:

| D | C | SID | DName | CID |
| :--- | :--- | :--- | :--- | :--- |
| CS | 101 | 11 | CS | 101 |
| CS | 101 | 11 | Math | 101 |
| Math | 101 | 11 | CS | 101 |
| Math | 101 | 11 | Math | 101 |
| CS | 101 | 12 | CS | 101 |
| CS | 101 | 22 | CS | 101 |
| CS | 101 | 22 | CS | 102 |
| Math | 103 | 33 | Math | 103 |
| Math | 103 | 33 | EE | 102 |
| EE | 102 | 33 | Math | 103 |
| EE | 102 | 33 | EE | 102 |
| CS | 102 | 22 | CS | 101 |
| CS | 102 | 22 | CS | 102 |

Then apply $\sigma_{D N a m e \neq D}$ or $C I D \neq C\left(\rho_{(D, C, S I D)}(\right.$ Enrolled $) *$ Enrolled $)$, which selects from the previous result only the rows for which Dname $\neq D$ or $C I D \neq C$ :

Tip: You can view selection as choosing tuples for inclusion, or choosing tuples for elimination by negating the $\theta$ condition. By DeMorgan's Law $\neg \theta$ is Dname $=D \wedge C I D=C$.

| D | C | SID | DName | CID |
| :--- | :--- | :--- | :--- | :--- |
| CS | 101 | 11 | Math | 101 |
| Math | 101 | 11 | CS | 101 |
| CS | 101 | 22 | CS | 102 |
| Math | 103 | 33 | EE | 102 |
| EE | 102 | 33 | Math | 103 |
| CS | 102 | 22 | CS | 101 |

From that result we project the SID attribute by applying $\pi_{S I D}(\ldots)$, which gives us:

| SID |
| :--- |
| 11 |
| 22 |
| 33 |

From there you can easily see that we have the $S I D$ s of all the students enrolled in two or more courses, which we natrual join with Student so we can project the SNames for the final result.

